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Math-odology: Methodology for teaching mathematics and tools for the implementation of virtual classes

Math-odology: Una metodología para enseñar matemáticas y herramientas para la implementación de clases virtuales

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Abstract Due to the COVID-19 pandemic that is affecting worldwide, the educational centers have been forced to change its regular classroom classes approach towards a virtual environment. This change has brought challenges and difficulties for teachers and students since this is a learning style that has not been used before for most of them. The area of Mathematics is a clear example of these challenges, since it is usually considered one of the most challenging subjects, and one that conventionally requires a classroom environment to be taught appropriately and pedagogically. Therefore, it is important to create a methodology for implementing virtual classes in Mathematics and also, search for tools to improve and enhance the learning process, while making it adaptable for the continuously changing learning environments. The proposed methodology is based on the study of pedagogical techniques, different Learning Management Systems (LMS), and online resources involved with the virtual teaching of Mathematics. To develop it, the Analysis Design Development Implementation Evaluation (ADDIE) method has been applied, while the illustration has been made using the Software Process Engineering Metamodel (SPEM), which allows developers and group members to adopt processes in a common language. This methodology was evaluated through a study case where

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a group of Mathematics university instructors were asked to apply the methodology in a class, while, another group was asked to impart a virtual class without it. Then, a questionnaire about their perceptions and expectations was applied to the students; and to the instructors about perceptions. Finally, after comparing the results it was concluded that Math-odology is highly applicable to adapt a mathematics class to a virtual environment. Also, it is necessary to link each activity with the right tools that contribute positively to the teaching-learning process.
Keywords e-learning tools, mathematics, methodology, online education.

Resumen Debido a la pandemia de COVID-19 que está afectando a nivel mundial, los centros educativos se han visto obligados a cambiar su enfoque de clases regulares en el aula hacia un entorno virtual. Este cambio ha traído desafíos y dificultades para profesores y estudiantes, ya que este es un estilo de aprendizaje que, la mayoría de ellos, no ha utilizado antes. El área de Matemáticas es un claro ejemplo de estos desafíos, ya que generalmente se considera una de las asignaturas más desafiantes, y una que convencionalmente requiere un ambiente de aula para ser enseñado de manera adecuada y pedagógica. Por tanto, es importante crear una metodología para la implementación de clases virtuales de Matemáticas y también, buscar herramientas para mejorar y potenciar el proceso de aprendizaje, haciéndolo adaptable a los entornos de aprendizaje en constante cambio. La metodología propuesta se basa en el estudio de técnicas pedagógicas, diferentes Sistemas de Gestión del Aprendizaje y recursos en línea relacionados con la enseñanza virtual de las Matemáticas. Para desarrollarlo se ha aplicado el método de diseño instruccional interactivo ADDIE (Análisis, diseño, desarrollo, implementación y evaluación), mientras que la ilustración se ha realizado utilizando el Metamodelo de Ingeniería de Procesos de Software (SPEM), que permite a los desarrolladores y miembros del grupo adoptar procesos en un lenguaje común. Esta metodología fue evaluada a través de un caso de estudio, donde se solicitó a un grupo de profesores universitarios de Matemáticas que aplicaran la metodología en una clase; mientras que a otro grupo se le pidió que impartiera una clase virtual sin el método. Con el objetivo de evaluar las percepciones y expectativas de los estudiantes en relación al método; y las percepciones de los profesores, se aplicó un cuestionario al finalizar la clase. Finalmente, luego de comparar los resultados se concluyó que Math-odology es altamente aplicable para adaptar una clase de matemáticas a un entorno virtual. Asimismo, es necesario vincular cada actividad con las herramientas adecuadas que contribuyan positivamente al proceso de enseñanza-aprendizaje.
Palabras Claves educación en línea, herramientas de e-learning, matemáticas, metodología.

1 Introduction

On January 30th, 2020, the novel coronavirus outbreak was declared as a Public Health Emergency of International Concern (PHEIC) by the World Health Organization, WHO (2020); this means the highest alarm there is. "A pandemic is defined as a large-scale outbreak of infectious disease that can significantly increase morbidity and mortality over a wide geographic area. Also, it causes significant economic, social, and political disruption" (Madhav et al., 2017, p. 315). "The field of education is not excluded from the pandemic dire effects. The COVID-19 has created the most massive disruption of education systems in history; it affects around 1.6 billion learners in more than 190 countries" (United Nations, 2020, p. 2).

In March 2020, at the height of the COVID-19 pandemic, at least one and a half billion students worldwide were forced to engage in remote learning (United Nations, 2020). Ecuador was not the exception; the education system has had to reinvent itself and adapt to the virtual modality, even though the Ecuadorian teachers and students were not prepared (Asanov, Flores, Mckenzie, Mensmann, & Schulte, 2020). However, despite the benefits of electronic classrooms and technological advances, many university instructors do not fully use them. One reason why they do not exploit the technical benefits is their unfamiliarity with specific applications and tools for virtual teaching. Additionally, instructors may not understand how the various web tools can assist them in planning the particular courses they teach (Asanov et al., 2020).

Likewise, according to the United Nations Educational, Scientific, and Cultural Organization, UNESCO (2020), the impact of the pandemic on the university higher education sector was significant, and the change in teaching modality was not received positively. Part of the disaffection stems from the fact that the content offered was never designed within a distance higher education course. Instead, it tried to make up for the absence of face-to-face classes with virtual classes without further preparation .

Virtual education is present in most high education institutes. It is difficult to find one that does not have a virtual campus and a virtual classroom for each subject, as an extension of the physical classroom. However, in practice, the ability of each instructor to continue teaching online largely depends on their experience in that regard. It must be considered that those subjects which include the development of professional competences through practice (e.g., clinics, engineering, science, and all those heavily dependent on practical training) are a source of more significant uncertainty, which will lead to a set of different impacts on the university. Finally, it is crucial to consider that mathematics generally present more difficulties in the virtual adaptation process (United Nations Educational, Scientific, and Cultural Organization, UNESCO, 2020).

Therefore, to address this gap, in this paper is first presented Math-odology, a methodology for implementing virtual classes in mathematics, considering the continuously changing of learning environments. Also, some information technology tools to be considered for the Mathematics instructors. Then, the case study is divided into two stages. On the one hand, an instructor prepares and imparts class to 13 students using their daily method. Then, another instructor uses Math-odology to prepare and address a class of 17 students. The results present the impressions from students about the course through a survey and a statistical analysis. Finally, a summary of the instructor's perceptions when using Math-odology.

2 Background

Pedagogy is a multidisciplinary science that fuses the areas of philosophy, psychology, anthropology, sociology, medicine, economics, and history. The contribution made by each of these fields of study favors the pedagogical task and provides the scientific bases that give the character of science to pedagogy (Baumfield, 2011). Pedagogy is continuously applied in the teaching-learning processes. It is responsible for studying and analyzing educational phenomena to provide systematic and intentional solutions that allow the teacher to guide their educational and training actions in the light of a set of principles, methods, practices, specific ways of thinking, and models.

2.1 Pedagogical practices and techniques

The daily pedagogical activities encourage the professor to find the necessary ways to improve the quality of teaching. The student-content relationship is established through some resource or didactic material, which approximates the student's access to observation, research, or understanding of reality (Baumfield, 2011). The pedagogical techniques qualify the teaching work due to they are in constant relationship with the personal characteristics and professional abilities of the teacher and other elements such as the characteristics of the group of students, the learning environment, the content to work, and the time available (Conole, Dyke, Oliver, & Seale, 2004). The choice of one or another technique will depend, among other factors, on the learning modality, be it face-to-face or virtual (Donnelly, Harvey, & O'Rourke, 2010). An effective instructor is who evolved from the role of teacher to being a facilitator. It despite influenced pedagogical practices in an era marked by online hyperactivity (Bailey & Card, 2009). Thus, pedagogical representation models are required; these incorporate diverse educational values and include patterns of action that raise awareness of the educational dilemmas surrounding the innovation (Gros, Suárez-Guerrero, & Anderson, 2016). Bailey and Card (2009) propose some effective pedagogical practices for effective online teaching, i) fostering relationships, ii) engagement, iii) timelines, iv) communication, v) organization, vi) technology, vii) flexibility, and, viii) high expectations.

2.2 Learning management systems

Due to the technological environment, it has been necessary to seek the application and use of technologies to improve the teaching-learning process. With this in mind, there are several Learning Management Systems (LMS) available for teachers and students alike, which provide a great range of digital academic resources (Juan, Steegmann, Huertas, Martinez, & Simosa, 2011). Said LMS are applications where the most common e-learning functions converge (Ninoriya, 2011). According to Ninoriya (2011), LMS' primary functions are to centralize the information and data from the classes and students, deliver teaching content rapidly and personalize content. In general, LMS could be classified into two groups: free and commercial. Each one has its own set of advantages and disadvantages, being the preferred ones the open applications. Nevertheless, this type of LMS has more options than commercial products. Among the most used on the free category are Moodle, Claroline, and Dokeos, while in the commercial kind, there are Blackboard and WebCT (TeachThought, 2016).

2.3 Online resources

Many tools and resources could be used by the students and teachers alike to engage and improve the learning process to aid in virtual teaching. Watfa and Audi (2017) presented that technology is essential for the teaching-learning process of mathematics because it improves it and offers more options to support the learning. Due to the widespread use of technologies and online resources available on the web, it is essential to focus on the mathematics resources (Bueno-Ravel & Gueudet, 2009). Among the mathematics specific online resources, the most used by students and teachers are Wolfram Alpha, Photomath, Geogebra, among others (TeachThought, 2016). These resources' main characteristic is that they allow the students to interact with mathematics concepts through graphics and the step-by-step resolution of problems. Teachers need to use these resources and the students to emphasize its importance as a complement of the learning, not as the only tool.

3 Math-odology

This section presents the methodology for planning mathematics into a virtual environment; and the tools for its implementation, which is called Math-odology. This methodology was developed using the Software Process Engineering Metamodel (SPEM 2.0) (Menéndez & Castellanos, 2015). Figure 1 presents the main steps for the method, which has been divided into four activities: i) Select learning methodology, ii) Selection of information to be imparted, iii) Selection of teaching strategy aligned with the learning methodology, iv) Selection of IT tools to comply with the plan. In the following sub-sections, each of those steps will be explained and giving special attention to step 4, which is one of the most critical parts of the contribution of this paper.

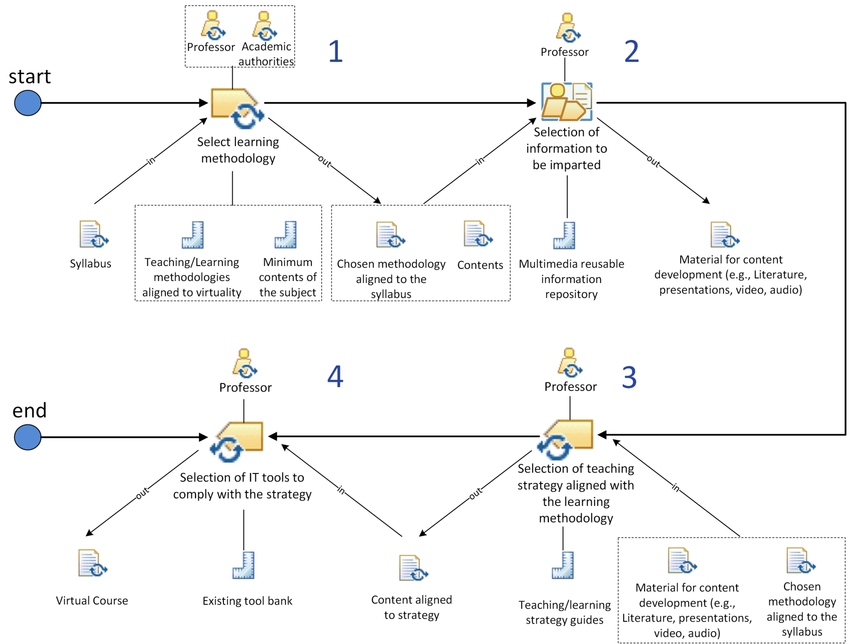

Fig. 1: Math-odology methodology
Source: Own creation

3.1 Select learning methodology

The first step is to select the appropriate learning methodology for the content that will be taught. The first required document is a guide with recommended teaching-learning methodologies, presented in Table 1 , which have to be aligned with the virtual method of teaching, preferring the student-centered rather than teacher-centered methods (Teachers.com, 2019). Also, the minimum contents of the subject and the syllabus are available. Also, professors need to comply with these documents for the class, provided by academic directives. The results of this stage are the selected methodology and the contents used in the planning of the course.

3.2 Selection of information to be imparted

For this step, the inputs are the chosen methodology aligned with the syllabus and the contents. The professor selects the information to be imparted, aiming to gather all the material referring to topics and subtopics from past courses such as books, documents, videos, presentations, audios.

Table 1: Teaching-learning methodologies.

Methodology	Characteristics
Direct instruction	The traditional method centers on the exposition and demonstrations of the topic from the instructor.
Flipped classroom	The traditional elements of the lesson taught by the instructor are inverted. The students study the primary educational materials at home, and then they are reviewed in the classroom.
Inquiry-based learning	It is based on student investigation and hands-on projects; the instructor is seen as a supportive figure who provides guidance and support for students throughout the learning process
Project-based learning	Students engage in an in-depth study of topics that impact their environment. It includes multiple content areas with the objective that the student appreciates how problem-solving can happen in the real world.
Game-based learning	Students work on missions to accomplish a defined goal, which is the learning objective. This objective is achieved by choosing actions and experimenting along the way. Likewise, in video games, according to progress, students earn points and prices.

Source: Own creation with data taken from
`https://teach.com/what/teachers-know/teaching-methods/#top`

This material must be aligned with the theme to be taught in class. The professor's task is to gather this information to create the new content adjusted to the new course format, prioritizing digital dissemination. As recommended by K. Vlasenko, Chumak, Lovianova, Kovalenko, and Volkova (2020), the materials for an online course can be video, animation or audio files, schemes, images, graphics, tables, computer simulators, presentations, and others (Abbasian & Sieben, 2016; El-ahwal & Shahin, 2020; K. V. Vlasenko et al., 2020). Then, as a result of this process, audio-visual content or material for content development is obtained to support the virtual mathematics course.

3.3 Selection of teaching-learning strategy linked to the methodology

Once the methodology, contents, and information to be imparted on the class are selected, it is necessary to define the teaching strategy used. That complies the best with the contents and methodology previously chosen. Moreover, it is crucial to consider that the selected strategy needs to be active and engage the student in the learning process.

3.4 Selection of IT tools to comply with the strategy

There are different alternatives to teach classes online, each with their pros and cons. However, it is essential to have a good platform for videoconferencing and complementing tools to teach sessions remotely. Thanks to this type of resource, the concept of “online, face-to-face university” has begun to be used. This concept allows students to study from home. Still, also it is possible to receive classes live, to interact with other participants, view presentations or videos, and engage with resources in work-groups. Therefore, to address the activity of the Selection of IT tools to comply with the strategy, below are presented several tools that permit to have a good performance during the lectures through the screen, focusing on math classes. These tools are shown on the resource Existing Tool bank (Figure 1).

The classes online courses have a significant impact by using visual resources like video, which aids in learning at the student pace (K. Vlasenko et al., 2020; Suduc, Bîzoi, Gorghiu, & Gorghiu, 2012; Suduc, Bizoi, & Filip, 2010). Moreover, some analysis of the content created for online courses highlights the importance of i) distribute the labor intensity of learning students activities correctly by weeks, providing interaction among the participants (Wrigley, Mosely, & Tomitsch, 2018) and ii) integrate various web tools and resources for learning the course and material (Jönsson, 2005).

A survey presented by Hart (2020) shows the two hundred most essential tools for learning, divided into three categories: i) Personal Learning (PL), ii) Workplace Learning (WL), and iii) and Education (E). According to this survey, the essential tool is YouTube concerning PL and E, and Zoom about WL. In the Appendix presented below, Table A1 presents a summary of the most representative tools of this survey, involving online classes.

Also on the Appendix section, on Table A2, it is presented a list of specialized platforms and interactive tools used for teaching mathematics.

Moreover, beyond the presented tools, Capterra (Gartner, 2020) is also presented. It is an online resource founded by Ryan Yeoman that has been available since 1999. It allows software buyers to find adequate software depending on the field they are going to be used. This web portal offers user reviews that have been validated and independent research in hundreds of software categories and the complete information on the costs of the licenses and subscriptions. It also allows the user to compare any software or application with similar ones, thus permitting the user to decide between several options and the reviews of other users, synthesizing so the pros and cons of the selected tool (Gartner, 2020). Finally, after choosing the tools, the professor can impart lectures aligned to the methodology and start with the virtual course for teaching mathematics.

4 Evaluation and results

For a first approach to the evaluation of the proposed methodology, a case study was performed. The math teachers prepared a class using the methodology, and also without it. Then, each class was taught to a group of first-year students of Mathematics I in the school of International Relations of Universidad del Azuay in Cuenca, Ecuador. After the class finished, a questionnaire was applied, which consisted of five questions about their experience with the session, their expectations for future classes, and understanding of the presented topic.

The survey was conducted on two student groups, Group 1 was made up of 13 students and received a class prepared without the proposed methodology, Group 2 was made up of 17 students and received the class designed with the Math-odology. The objective of the survey was to measure the degree of satisfaction of the class received, and it was made up of five questions; a Cronbach's alpha of 0.875 was obtained, which reflects an adequate level of reliability.

Regarding the tools used during the class, it is evident that there is greater acceptance in the group that used the proposed methodology (see Table 2). The Mann-Whitney U value was 63,500, and the p-value is 0.025, so it is concluded that the degree of agreement on the use of the tools used by the teachers differs in both groups with a risk level less than 5% (p-value is less than 0,05).

Table 2: Tools used during the class.

Do you consider that the tools used during the class were useful to understand the mathematics concepts taught?	Group 1. Without methodology		Group 2. With methodology	
	n	%	n	%
1. Strongly disagree	0	0	0	0
2. Disagree	2	15,38	0	0
3. Neither agree nor disagree	2	15,38	1	5,88
4. Agree	4	30,77	3	17,65
5. Strongly agree	5	38,46	13	76,47

Source: Own elaboration based on the results obtained from the evaluation

The students who received the class with the methodology maintained their level of attention to a greater extent than those who received a prepared class without the proposed solution (see Table 3). The Mann-Whitney U value was 53,000, and the p-value is 0.008, so it is concluded that there are statistically significant differences between the two groups with a significance level greater than 95%.

Concerning the expectations of the class, it was found that the value of the Mann-Whitney U was 54,500, and the p-value was 0.006. Therefore, it is concluded that there are statistically significant differences between both groups with a level of significance greater than 5%. It also evidences a greater acceptance in the group that used the proposed methodology (see Table 4).

Table 5 shows the degree to which students in both groups want to teach virtual classes to be used by other teachers. The Mann-Whitney U value was 35,500. The

Table 3: Level of attention during the class.

Did the given class not capture your attention from start to finish?	Group 1. Without methodology		Group 2. With methodology	
	n	%	n	%
1. Strongly agree	3	23,08	0	0
2. Agree	1	7,69	0	0
3. Neither agree nor disagree	2	15,38	2	11,76
4. Disagree	3	23,08	2	11,76
5. Strongly disagree	4	30,77	13	76,47

Source: Own elaboration based on the results obtained from the evaluation

Table 4: Expectations of the class.

Did the imparted class meet your expectations?	Group 1. Without methodology		Group 2. With methodology	
	n	%	n	%
1. Strongly disagree	0	0	0	0
2. Disagree	2	15,38	0	0
3. Neither agree nor disagree	3	23,08	0	0
4. Agree	3	23,08	3	17,65
5. Strongly agree	5	38,46	14	82,35

Source: Own elaboration based on the results obtained from the evaluation

p-value is 0.000, so it is concluded that there are statistically significant differences between both groups with a significance level greater than 95%. It is evidenced by a greater acceptance in the group that used the proposed methodology.

Table 5: Desire for other teachers to teach the class in a similar way.

Would you like this teaching style for virtual classes to be used by other teachers?	Group 1. Without methodology		Group 2. With methodology	
	n	%	n	%
1. Strongly disagree	1	7,69	0	0
2. Disagree	1	7,69	1	5,88
3. Neither agree nor disagree	3	23,08	0	0
4. Agree	5	38,46	0	0
5. Strongly agree	3	23,08	16	94,12

Source: Own elaboration based on the results obtained from the evaluation

Regarding the used strategy, the students who received the class prepared with the methodology consider to a greater extent than it is appropriate for their understanding of the topic (see Table 6). The Mann-Whitney U value was 75,000. The p-value is 0.096, which is why it is concluded that there are no statistically significant differences between both groups with a significance level of less than 95%.

As for the perceptions of the teacher, they state that the methodology is useful when planning a mathematics class that has to be taught online, which was previ-

Table 6: The strategy used and understanding of the topic.

Is the strategy used by the teacher to teach mathematics classes appropriate for their understanding of the subject?	Group 1. Without methodology		Group 2. With methodology	
	n	%	n	%
1. Strongly disagree	1	7,69	0	0
2. Disagree	1	7,69	0	0
3. Neither agree nor disagree	3	23,08	0	0
4. Agree	2	15,38	6	35,29
5. Strongly agree	6	46,15	11	64,71

Source: Own elaboration based on the results obtained from the evaluation

ously imparted regularly. It was a difficult task to adapt since the beginning of the COVID 19 pandemic and the change to virtual education because they have never done it before. The methodology allowed them to have a structure when planning the class, starting from the choosing of a methodology that is aligned with the virtual reality of education nowadays. The most critical step for each of the teachers is the tool bank provided for the exercise to find online resources and tools that were useful for the class. These resources have not been previously used by the teachers, except Google Drive for sharing documents. They consider that the methodology is useful and could be used to plan other classes; they only recommend using more enabled tools.

5 Conclusions

After developing the Math-odology and based on the results of the case study, it has been shown that it is highly applicable and useful for the planning of virtual classes of Mathematics for professors who have little or no experience teaching in an online environment. The tool bank with resources adaptable to be used in a mathematics class is an essential complement for the methodology, according to the professors who applied it. It was also found that the students had an acceptable response to the class planned with the method and had an excellent adaptation to online resources.

The survey's objective was to evaluate the students who participated in the class after it. The said questionnaire had five questions, and they had an adequate level of reliability. The students from Group 2, who participated in the applied methodology class, considered that the tools used during the class were useful, they were able to engage with the topic, their expectations were met, and they recommend that other professors apply a similar method for their classes.

It is important to note that even though the results were acceptable from both groups, the statistical difference among them shows that the second group was more optimistic, adept, and motivated by using the methodology in the class.

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Appendix

Table A1: Most essential tools for Learning and Education.

Category	Description	Featured Tools
Learning Management System (LMS) Platform	The LMS is widely used e-learning tools that improve the quality of lessons, students learning experience, and the construction of their understanding of specific topics (Ninoriya, 2011), (Kasim & Khalid, 2016), (Syaad & Hidayat, 2018).	Google Classroom Moodle Canvas
Video Meeting Platform	The Video Meeting Platforms are nowadays the most used tools for virtual meetings between two or more participants at different sites. These tools facilitate initiating and conducting live conferences by using computer networks to transmit audio, video, and text data.	Zoom Google Meet Adobe Connect Cisco Webex
Collaboration Platform	The Collaboration Platforms are virtual workspaces where the tools and resources are centralized to facilitate personal interaction and communication within a class.	Microsoft Teams Slack Trello Padlet SharePoint
Online Whiteboard	An Online Whiteboard is a virtual representation of a learning space or whiteboard where the professor and students can interact in real-time.	Miro Jamboard MS Whiteboard Mural
Chat Tool	The Chat Tools are the most used to interact with people by using text. Thus, these are relevant tools to communication between professor and students	WhatsApp Skype Telegram
Resources	The Resources represent the virtual infrastructures contain information for learning and teaching (e.g., videos, books, lectures).	YouTube Wikipedia Vimeo TED talks Kindle
File-Sharing Tools or Platforms	The File Sharing tools or platform represents the environments where information about the course could be located and shared between students and professors.	Google Docs Dropbox One Drive We Transfer

Source: Own creation with data taken from Ninoriya (2011); Kasim and Khalid (2016); Syaad and Hidayat (2018)

Table A2: Interactive tools and platforms for teaching mathematics.

Tool	Description
Wolfram Mathematica	Wolfram Mathematica is an online program used in scientific, engineering, mathematical, and computational areas (Wolfram, 1991).
Geogebra	Geogebra brings the resources for online courses and the feasibility to create, share, and modify math models and simulations (GeoGebra, 2020).
Photomath	Photomath is a free mobile app that can read and solve mathematical expressions using the smartphone camera. It presents the final result, along with a step-by-step explanation (Photomath, 2020).
Mathway	Solves any mathematics expression by taking a picture of it or enter it manually. This app only gives the final result; it is necessary to upgrade to premium (Mathway, 2020).
Socrative	Socrative is a cloud-based student response system. It allows teachers to create simple quizzes that students can take quickly on any technological device (*Home - Socrative*, 2020).
Kahoot	Kahoot is a game-based learning platform that makes it easy to create, share, and play learning games or trivia quizzes in minutes (*Welcome back to Kahoot! for schools*, 2020).
Quizziz	Quizziz is a solution that allows conducting student-paced formative assessments in a fun and engaging way for students (*Quizizz – Free Quizzes for Every Student*, 2020).
My Math Lab	MyMathLab is an online interactive and educational system, which covers many levels of mathematics. Some features are homework, quizzes, tests, full eText, and multimedia tutorials. For professors, it allows customizing the course, records evaluation, and time spent of each student (Law, Sek, Ng, Goh, & Tay, 2012).

Source: Own elaboration with data taken from the websites of each tool: Wolfram (1991); GeoGebra (2020); Photomath (2020); Mathway (2020); *Home - Socrative* (2020); *Welcome back to Kahoot! for schools* (2020); *Quizizz – Free Quizzes for Every Student* (2020); Law et al. (2012)